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# The physical basis of scoring athletic performance

By Basil Grammaticos

ABSTRACT

*This article sets out to prove that it is possible to create scoring tables for the widest possible range of performance in athletic events. The author, a physicist who now works in the modelling of biological phenomena, describes a new scheme, based on a simple analytic expression. The scheme's construction is based on two main ideas. The first is that the number of points attributed to a given performance should increase with the (useful) energy expenditure of the athletic effort. The second, inspired by the works of Dale Harder, is that the number of points should increase as the number of athletes who have realised a particular performance decreases. The two ideas converge with a proposal for a scoring system that closely approximates that currently used by the IAAF for the combined events except (by construction) for the lowest performances.*

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## Introduction

Competition, typically for scarcely available resources, is hardwired in all living organisms. In human society the drive for competition has led to various ritualised activities, the most prominent of which are sports. While recreation sports express the need for physical exercise and are essentially of ludic character, competitive sports focus on the comparison of the individual (or team) to others within a precise framework of rules and practice. The notion of comparison is even more deeply ingrained in the sports with a quantitative character, where the performance can be objectively assessed. The advantage of codified, quantitative sports is that they allow universal comparisons: the performance of two athletes can be compared even though they have never competed against each other. On the other hand, it would be illusory to extend these comparisons to non-contemporary athletes: too many parameters change over time (material conditions, rules, scientific support, etc.) to make diachronic comparisons meaningful. Still, in sports where continuity is ensured through a sustained activity, the existence of records and the monitoring of the performances is quite useful since it, almost always, involves short time steps, typically of the order of a few years.

While quantitative measurements allow comparisons of the performance of competitors within the same discipline, the situation becomes more complicated when different disciplines are involved. Clearly, such a comparison can be meaningful. Everybody would agree that the performance of a world champion in some event is better than that of an inexperienced competitor in a different event. But there is a possibility to make this comparison more precise and to distinguish performances on which only specialists could *a priori* pronounce themselves. The need for this comparison goes back to the trait of competition. But here exists also another, more practical, necessity: that of the combined events. Since the most ancient times, the quest to find the best all-around athlete has led to the proposal of combined or multi-events. While classification in the ancient Greek pentathlon was based on competitive rankings in four events with the result of the fifth event, wrestling, being the tie-break, the modern approach is much more quantitative[1]. With the combined event scoring tables, performances are converted to points allowing thus a fine scoring for each of the disciplines and greatly facilitating inter-event comparisons[2].

In this paper, we shall examine the question of scoring from two points of view. The first focuses on the physical (dynamic, physiological) basis for scoring[3]. The second is inspired by the theories of Dale HARDER (2001) (his "apples to oranges" approach), which furnish a basis for inter-sport comparisons[4]. We shall compare HARDER's tables to those currently used by the IAAF[5] and establish their close parallel. Finally, our two points shall converge with a proposal for a scoring system based on a simple analytic expression satisfying the basic physical requirements, closely approximating the current IAAF tables and carrying the ideas of HARDER to their logical extrapolation. Such a system is not meant to be a substitution of the IAAF tables but rather a "proof of concept" that one could develop tables that would score performance over the widest possible range of performance.

## The energetic basis of scoring

How does one go about setting up scoring tables? Two things are needed. First, one must decide on the correspondence between performance and points within a given discipline and second on the correspondence of performances across disciplines. The commonly used approach relies heavily on statistics and, provided a large data basis exists, it gives fairly reliable results[7]. Scoring tables are obviously not written in stone and have to be revised regularly (although not too often, lest they destabilise the combined events community). In particular, whenever a major change, due to equipment evolution or rule changes, intervenes, the tables must be adapted in order to re-establish a fair comparison.

While the statistical approach to scoring table construction gives perfectly acceptable results, it is rather phenomenological and does not lead to an understanding of the underlying mechanism. In the following paragraphs, we will propose an *ab initio* approach, where the assumptions for the table construction will be clearly stated and which will make possible in fine a very simple parameterisation.

In my modelling courses, I always ask the question: "what is the physical equivalent of money?" Eschewing all Marxist overtones, we can safely answer that the physical quality that is the best candidate is free energy, i.e. the amount of energy that can be converted into work. Extrapolating the situation in a sport setting, we can decide that the reward of a performance, in terms of points attributed, must be closely related to the work necessary in order to produce the performance[8]. It is thus of capital importance to know the energetic cost of the various disciplines[9].

A caveat is in order at this point. It is clear that we will limit our discussion to individual sports where the performance is essentially conditioned by the energy produced. Sports like athletics or swimming, which depend mainly on physiological factors, fit easily into this frame. Sports where the performance is not just a matter of physiology but where aesthetic and precision factors enter largely could also be included in a generalised approach. The notion of free energy and its relation to entropy would have to be clearly defined for these disciplines. This is quite a formidable task and nobody, at least to my knowledge, has addressed this question.

In the case of athletics, which will be the focus of the present study, the energetic costs of the various events are well-known and easily assessed. In the field events the measured length s is proportional to the (useful) work produced i.e. $\epsilon=\lambda s$. In the case of running, the measurement of the energetic cost has been the object of detailed studies. Following the analysis of the literature, we may well assume that the energy expenditure Â when running a given distance with mean velocity v is given by:

$$\epsilon=\alpha+\beta v+\gamma v^2 \qquad (1)$$

where $\alpha$, $\beta$, and $\gamma$ are constants. The precise values of these constants are not important but one point is remarkable: the coefficient $\gamma$ is very small, typically several hundred times smaller than a. Consequently, we expect this term to play a role only for the shortest races, where the registered speeds are in excess of 10m/s. Thus, a good approximation of the energetic cost of running could be $\epsilon=\alpha+\beta v$, i.e. the energy is a linear function of the velocity. (This is definitely not true in other disciplines in which either higher velocities are involved, like cycling, or higher medium resistance, like swimming. There, the energetic cost is definitely not a linear function of the velocity)[10].

A (very) simple model of a scoring table would thus be one where the number of points p is a linear function of the velocity v for running or of the distance s for field events. The relation of points to velocity or distance now becomes:

$$p=a+b\epsilon \quad (2)$$

where $\epsilon$ stands for v or s. The current IAAF combined events tables follow this model fairly well, in particular in their upper (above 300 points) part[6]. In order to investigate a possible effect of quadratic velocity term, we have performed a fit of the IAAF tables for the100m event over the range 300-1300 points, using either of the two expressions $p=a+bv+cv^2$ or $p=a+bv$. In the first case we obtained the values of parameters a=-2007.75, b=259.85, c=6.25 (in the appropriate units, the velocity being expressed in m/s) with a $\chi^2$ of 0.035. In the second case we found a=-2500.4, b=371.3 with a $\chi^2$ of 0.15. Given the very small values of $\chi^2$, both fits are excellent. Using these formulae to compute the number of points for a velocity v=9.70m/s, we obtain 1101 for both of them (the difference lying in the decimal points, which are rounded down to the next lower integer), the table value being 1103. Thus, the linear approximation is excellent even for very short races, where one expects the highest values of velocities.

The advantage of a linear expression like (2) is that it can be made dimensionless and adapted to tables of arbitrary extension. Rewriting (2) as:

$$p=p_0(\epsilon/\epsilon_0{}^{-1}) \qquad (3)$$

we can immediately grasp the meaning of the parameters. Here Â0 is the performance that gives precisely 0 points while $p_0$ is the number of points one obtains when $\epsilon=2\epsilon_0$.

While the "linear" tables are excellent and realistic, there exists still one point that, to the physicist's eyes, appears arbitrary: that of the cut-off $\epsilon_0$. Since the tables are meant essentially for competitors the reasons for this cut-off are obvious. Still, it remains that the natural cut-off should be "performance zero" i.e. v=0 or s=0. However reducing the scoring tables to an expression $p=a\epsilon$ would have been an oversimplification. We shall return to this point in the following sections.

## Harder's approach to scoring

HARDER has developed his approach to scoring in order to compare athletic achievements in different sports. The key to this comparison is that you "... compare the number of athletes who reach any given level ..." (proportional to the number of athletes competing in that sport, of course). The quantitative

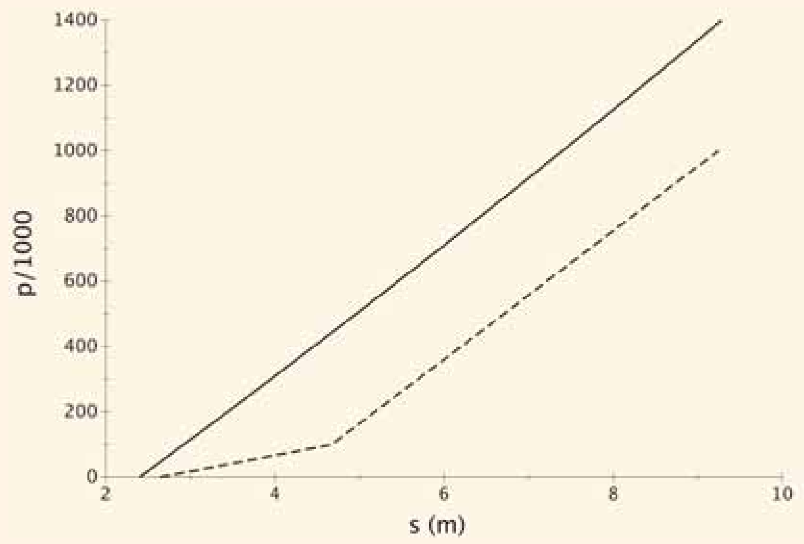


Figure 1: Correspondence between the length (s) in the long jump and points (p) in both IAAF (continuous line) and Harder's (dashed line) tables

basis of the method is the following. A mark of 100 points is attributed to some performance if a fraction of 0.5 of the population can realise a score equal or better than this. For 200 points, only a fraction of 0.05 of the population can do better than this performance. The next 100 points, i.e. 300, correspond to a performance realised by just 0.005 of the population and so on up to 1000 points where only a fraction $5.10^{-10}$ of the population can realise the corresponding performance.

We shall not pursue our analysis of HARDER's work here. It contains a monumental amount of data collection and statistical analysis combined with a solid knowledge of sports and an abundance of common sense. Moreover, the correlation of his tables for athletic vents with the IAAF tables is perfect. If we limit ourselves to points above 200, we obtain a correlation of practically 1 between the two. In Figure 1 we give the correspondence between the performance or length s in the men's long jump and points in both IAAF (continuous line) and HARDER (dashed line) tables. We remark that above 100 points both curves are linear, which explains the excellent correlation.

In what follows, we shall limit ourselves to the use of just the correspondence between points and percentile of population realising a performance as proposed by HARDER. It is a well-established statistical fact that the distribution of human performance in various domains follows a bell shaped curve. The details may vary but the fact remains that most individuals perform close to the median with only a very small percentage registering exceptionally good or bad performances. Given this, the fraction of the population $f(\epsilon)$ realising a performance better than some threshold $\epsilon$ (which is crucial in HARDER's approach) should be given by a step-like curve, which is just the integral of the bell-like curve on page 49. In Figure 2 we present graphically such a dependence. Next, we proceed to propose an analytic expression for this curve:

$$f(\epsilon)=(1+a)/(1+ae^{\epsilon})(4)$$

We have f(0)=1, i.e. 100% of the population realises some performance above or equal to zero. The symbol e is the basis of the natural logarithms. The parameter a can be related, for instance, to some performance $\epsilon_0$ for which we have f=1/2. We find $a=1/(e^{\epsilon}0-2)$, which

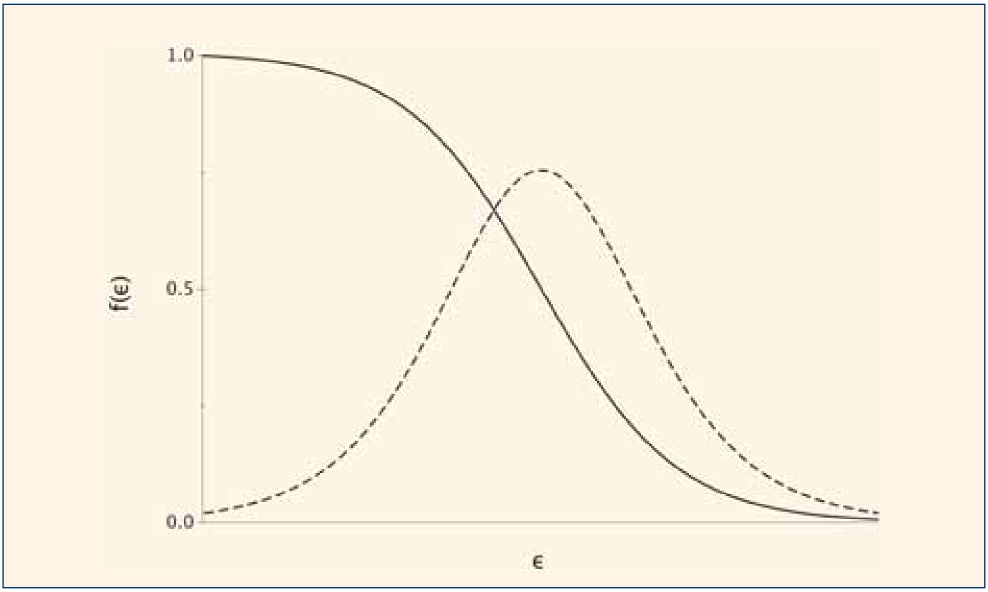


Figure 2: Fraction of the population ($f(\epsilon)$) realising a performance better than a given threshold ($\epsilon$) (continuous curve) and the distribution of performances (dashed curve)

allows us to rewrite (4) as $f(\epsilon)=(1+(e^{\epsilon}-1)/(e^{\epsilon_0}-1))^{-1}$. The expression (4) will be used in the following section for the introduction of a new scoring system.

## Proposal for a new scoring system

Re-examining Figure 1, we remark that HARDER's scoring of the performances of the lower 50% of competitors follow a different curve. This is an attempt in these tables to accommodate a large part of the population, 95% in fact. While this goes in the right direction, we believe that it is not quite sufficient. From our arguments based on energy expenditure, the zero in a scoring table that aims at universal applicability (and not just at competitors of a certain level as is the case of the IAAF tables) should correspond to zero performance.

Going back to the relation of points to performance and interpreting it in the spirit of HARDER, i.e. a progress of 100 points means that the fraction of the population realising it is divided by 10, we can introduce a logarithmic relation between points and performance, i.e. p is proportional to logf, where log is the decimal logarithm. More precisely, we propose the following scoring formula:

$$p=p_0 \ln(1+b(e^{c\epsilon}-1)) \qquad (5)$$

where we have opted for the use of the natural logarithm ln instead of the decimal one. Clearly, from (5), when $\epsilon=0$ we find p=0. When $\epsilon$ becomes large enough, the term 1-b is negligible compared to $be^{c\epsilon}$. Neglecting it completely we find that p becomes again a linear function of $\epsilon$. Thus, the curve of the number of points as a function of the performance starts from zero with a strong curvature and goes over to a practically linear segment.

In order to give a better idea of the behaviour of the expression (5) we present in Figure 3 the IAAF scoring for discus throwing together with the best fit obtained for this using only scores above p=200 in the IAAF tables. We remark that the fit is quite reasonable: the difference in points at around 50m where the two curves are farthest apart is less than 20 points. The values for the three parameters are $p_0$=236.173, b=0.43092 and c=0.0904 $m^{-1}$.

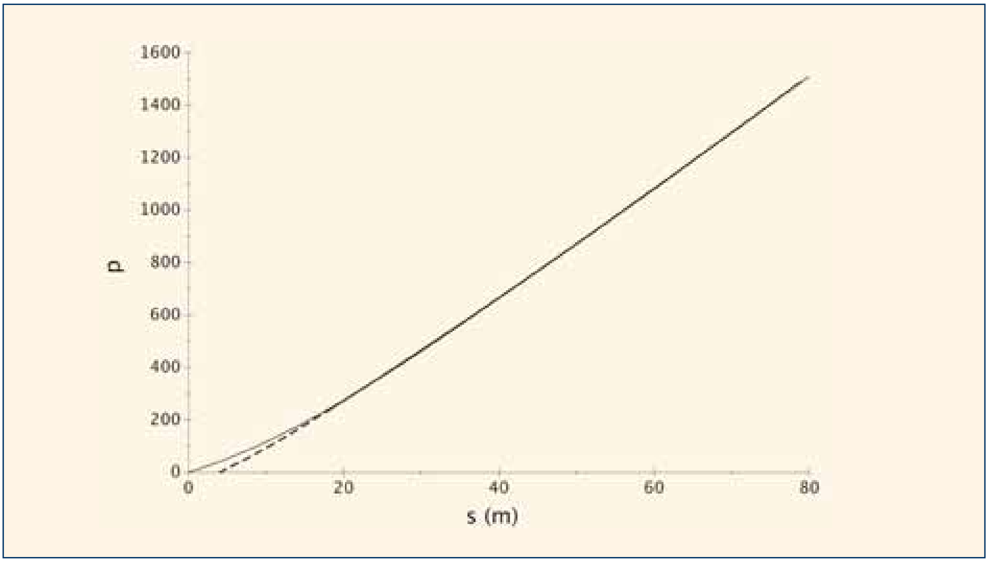


Figure 3: Points (p) as a function of the length of a throw (s) in the IAAF scoring table for the men's discus (thick curve, dashed below 200 points) and the best fit obtained with equation 5 (thin curve)

In any case, it shows that it is possible to score performance over the widest possible range. On a pure anecdotal level, the performance in men's discus throw that would score 1 point is just 7 cm.

## Conclusion and outlook

In this paper we have presented an approach to scoring based on physical considerations combined with HARDER's approach on how to compare performances in various sports based on population distributions. Our aim was to propose a simple analytical formula that would make computations straightforward. Moreover, from an energetic point of view, we felt that "everything should count" i.e. any valid performance should obtain some points all the way down to zero performance (which would obtain zero points). In our parameterisation, we have focused on the IAAF scoring tables and proposed a scheme that is in fair agreement to those tables from p~300 and above.

Since this is a physicist's approach, we are concerned about application to limiting cases, namely close to zero points, in the present situation. Several small problems seem to exist in the athletics rules as they stand. It is clear that for a hurdle race a competitor who cannot jump over hurdles, say somebody who would take minutes to complete the course, must push them down. However, deliberately knocking down hurdles is in principle not allowed by the rules. So, in these extreme cases the rule should be interpreted in a more permissive way. The style of triple jump for somebody seriously impaired should also be re-examined. In the shot put, the use of a stop board would makes performances under 11cm impossible. In such a situation, the possibility to remove the board should be examined. Of course, all these points (and several more) are not so important from a scoring point of view. However, they would have to be addressed for the sake of the coherence of the scoring system proposed.

The extension of our approach to other, "quantitative", sports is straightforward. Whenever the energetic cost of some sport is known and some analysis, a la HARDER, of the population distribution of the performance exists, it is quite easy to set up the corresponding scoring formula. Swimming (as well as the author's favourite, Fin-Swimming[11]), for instance, could be dealt with without additional difficulty.

Finally, and on a more speculative level, one can wonder which physical principles can be brought into play for sports that are of a qualitative character. Sports where the scoring of performance is based on the aesthetics of the motion would be particularly challenging and therefore most interesting. They would necessitate a precise biomechanical description of the "perfect", aesthetically pleasant motion[12]. Deviations from the latter should be considered as disorder, entropy increasing, processes and awarded penalties in a scale that will have to be proposed in order to agree with the current, human, expert judging. Although these ideas do put a tall order on physics in its relation to sports, we are optimistic that they could, if properly pursued, lead someday to a fair multi-sport scoring system.

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## NOTES

1. There exist a great many books on ancient Greek sport, several of which deal with pentathlon. The reader can for instance consult the following references:
   MILLER, S. G. (2004). *Ancient Greek Athletics*. Yale University Press.
   SANSONE, D. (1992). *Greek Athletics and the Genesis of Sports*. University of California Press.

2. IAAF Scoring Tables for Combined Events, 2001.

3. The literature on physics and its relation to sports is quite abundant. The following references are both classics in the matter:
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4. HARDER, D. (2001). *Sports Comparisons: You can compare apples to oranges*. Castro Valley, CA, USA: Education Plus (www.strengthopedia.com).

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6. TRKAL, V. (2004). The development of combined event scoring tables and implications for the training of decathletes, *New Studies in Athletics*, 18:4, 7-12.

7 The following reference explains in simple, understandable terms how to use statistics in order to establish equivalences between performances: MUREIKA, J. R.; COVINGTON, D. & MERCIER, D. (2000). The 1999 Mercier scoring tables: A how-to guide. Athletics: *Canada National Track & Field Magazine*, April/May 2000.

8. PURDY is probably the first to have applied a physics' inspired approach to the construction of scoring tables. Here are his original references:
   PURDY, J.G. (1970). Computer Generated Track Scoring Tables, *Medicine and Science in Sports*, 2, 152.
   PURDY, J.G. (1974). Least Squares model for the running curve, *Research Quarterly*, 45, 224.

9. The literature on the energetic cost of locomotion is overwhelming. The references below may serve as starting point for the interested reader:
   DANIELS, J. (1978). *Conditioning for Distance Running - The Scientific Aspects*, Wiley & Sons.
   KRAM, R. & TAYLOR, C. R. (1990). Energetics of running: a new perspective. *Nature*, 346, 265.
   DI PRAMPERO, P. E.; FUSI, S.; SPULCRI, L.; MORIN, J. B.; BELLI, A. & ANTONUTTO, G. (2005). Sprint running: a new energetic approach, *Journal of Experimental Biology*, 208, 2809.

10. DI PRAMPERO, P.E. (1986). The energy cost of human locomotion on land and in water. *International Journal of Sports Medicine*, 7, 77.

11. GRAMMATICOS, B. (1989). Construction and use of scoring tables for finswimming. *Finswimming Manual*, Ed. K-H. Kerll, Stuttgart: Verlag Stephanie Naglschmid, p. 108.

12. HAY, J.G. (1993). T*he Biomechanics of Sports Techniques*. Benjamin Cummings