Shot Put With Lighter Implements

NSA © by IAAF
29:1; 75-81, 2014

by Basil Grammaticos

ABSTRACT

Following a competition in 2012, a group of the world's top shot putters were invited to compete using a 5kg implement. The results were somewhat surprising in that the distances achieved were not as long as might be expected, based on the assumption that the length of a throw is inversely proportional to the mass of the shot. The aim of this paper is to provide an intuitive, physics-based interpretation of the results of the aforementioned competition and set a frame for the description of shot putting with implements of non-standard weights. The model, combined with the classical kinematics results for a projectile motion under the influence of gravity (and neglecting air resistance), allows the derivation of a simple expression for the dependence of the length of throw on the implement mass. This result is compared to existing performances and the limitations of the model when too heavy or too light implements are used are discussed. It is hoped that the results can assist coaches by providing a tool for interpreting the capability of athletes based on their results with different weight shots.

AUTHOR

Basil Grammaticos, Ph.D., is a Director of Research at the National Scientific Research Centre (CNRS) in Paris and is currently head of the modelling team of the joint University of Paris VII – University of Paris XI Laboratory for Imaging and Modelling for Neurobiology and Cancer Studies (IMNC).

Introduction

This paper has been motivated by the results of a 2012 competition[1], where top-class shot putters were invited, after the normal competition, to vie for the longest put of the year. Thus, after having completed an event using the regulation weight, 7.257kg shot, the athletes returned to the circle in order to compete with a lighter, 5kg implement. The results with the normal implement were the following:

Name of Athlete	Country	Distance with 7.257kg
Reese Hoffa	USA	21.72m
Tomasz Majewski	POL	20.84m
Dylan Armstrong	CAN	20.63m
Justin Rodhe	CAN	20.63m
Jacub Giza	POL	18.72m
Kamil Zbroszczyk	POL	18.19m

With the lighter one, the order did not change a lot and the throw distances were:

Name of Athlete	Distance with 5kg
Hoffa	25.20m
Rodhe	24.68m
Majewski	24.54m
Armstrong	24.36m
Giza	21.34m
Zbroszczyk	20.50m

Now, at first sight, these results may appear astonishing. The naive assumption that the length of the throw is inversely proportional to the mass of the shot would have one expect much longer throws with the lighter implement. However, the collected data disputes this proportionality argument forcing one to seek a better understanding of the situation. Therefore, the aim of this paper is to provide an intuitive, physics-based, interpretation of the results of the aforementioned competition and set a frame for the description of shot putting with implements of non-standard weight. In fact, contrary to what is alluded to in the title, our approach is valid also in the case where a heavier implement is used (more on this in the Discussion).

In what follows we start by presenting our model and then analyse the kinematics of the shot after it has left the thrower's hand. We combine these two items in order to make a prediction on the dependence of the throw length to the mass of the implement and compare it to existing data. Finally, we discuss the results obtained, pointing out limitations to their domain of validity and propose a somewhat better model.

The Model

It is beyond the scope of the present paper to present a biomechanically accurate model of the shot put. On the contrary, we shall over-simplify the shot putting process in order to isolate the most salient features, which, we believe, are the ones determining the length of the throw as a function of the mass of the implement.

In our model we distinguish two phases, which are fundamentally different. The first is what we call the acceleration phase. The thrower starts from a position where his velocity is zero and, using one of the two customary techniques, glide or spin, moves from the back to the front edge of the circle accelerating all the way. In the glide technique, there is a substantial vertical acceleration, while in the spin technique a centrifugal acceleration is perceived in the athlete's frame, but these details are not expected to play a crucial role and thus we shall not delve further on these points. The net result of the acceleration phase is to bring the hand of the thrower holding the shot to some velocity $\boldsymbol{v_0}$. Since the thrower is much more massive than the shot (typically 100kg compared to 4-7kg) a small difference in the mass of the shot will not make any difference when it comes to the value $\boldsymbol{v_0}$ of the velocity attained. Thus as a first approximation we can assume that the acceleration phase leads to a velocity $\boldsymbol{v_0}$ independently of the mass (***m)*** of the implement.

The second phase is that of the throw itself, during which the thrower pushes the shot and, expending a quantity of energy ***E***, which we take to be always the same, increases the kinetic energy of the shot from

$$\frac{1}{2}mv_0^2 \quad \text{to} \quad \frac{1}{2}mv^2$$

where *v* is the velocity at which the shot leaves the thrower's fingers. We have thus

$$E = \frac{1}{2}m(v^2 - v_0^2) \qquad (1)$$

It is clear from this expression that the final velocity of the shot depends crucially on its mass. The separation of the throwing process in two phases is quite a natural one and, in fact, common to all throws. In his book on throwing events, SILVESTER distinguishes precisely these two phases in his discussion of the biomechanics of throwing techniques[2]. Indeed, the corresponding chapter deals with the processes of "developing momentum in the run-up area" (a most restrictive one in the case of shot put) and of "transmitting energy from the body to the implement", corresponding to the acceleration and push phases of our model.

The Kinematics of Shot Putting

Once the shot leaves the hand of the athlete, its trajectory is subject to the usual laws of ballistics. The trajectory of a projectile released at height ***(h)***, with velocity ***(v)*** and at an angle (θ***)*** with respect to the horizontal is given, as a function of time ***(t)*** by the equations

$$x = v\cos\theta t \quad (2a)$$

and

$$y = h + v\sin\theta t - \frac{1}{2}gt^2 \quad (2b)$$

where ***x, y*** give the position of the (centre of mass of the) shot, measured along the horizontal and vertical axes and where ***g*** is the gravitational acceleration. These equations can be found in any elementary physics textbook. They are derived by neglecting the effect of air resistance on the moving body. In the case of the shot put, this is perfectly justified as shown by LICHTENBERG & WILLS[3]. The length (***L***) of the throw is given by the value of ***x*** when ***y*** = 0. Eliminating ***t*** between the two equations of (2a & b) we find that ***L*** is given by

$$gL^2 - 2v^2\cos\theta(\sin\theta L + h\cos\theta) = 0 \quad (3)$$

At this point it is advantageous to introduce non-dimensional quantities

$$S = \frac{L}{h} \quad \text{and} \quad d = \frac{v^2}{2gh}$$

Using typical values for ***v*** = 14m/s, ***h*** = 2m, and given that ***g*** = 9.8m/s^2, we find for ***d*** the value ***d*** = 5[4]: in the same spirit, a throw of ***L*** = 20m gives an ***S*** of 10.

The standard textbook answer as to the optimal angle in ballistics is θ = 45°. However this is valid only for throws from zero height. Since in our case the release height is not negligible with respect to the other parameters, we expect the optimal angle to be smaller than 45°. Still, in order to simplify our argument let us at first neglect this effect and assume θ = 45°. We find that the throw length is given by

$$S = d + \sqrt{d^2 + 2d} \quad (4)$$

this can further be approximated by

$$S = 2d + 1 \quad (5)$$

(the first correction being $-\frac{1}{2d}$, of the order of 1% given the typical value of ***d***)

We can now go back to the full angle dependence. The optimisation of the angle gives a solution around θ = 41° (using our standard values of ***v*** and ***h***). This is in agreement with previous results[5,6] but with slight disagreement with measurements based on competition data[4,7,8], which give angles around 37°- 38° . The explanation of this discrepancy is given in an elegant way by LENZ & RAPPL[9] who considered the correlation between the velocity of the shot and the release angle. Still, for the case at hand, given the small deviation of the optimal value of the angle from 45° and the fact that we use an approximation for ***S***, equation (5), we can safely neglect the angle effect and work just with the expression ***S = 2d + 1***.

The Dependence of Throw Length on Implement Mass

Starting from equation (5) we revert to quantities with dimensions and find for the throw length

$$L = \frac{v^2}{g} + h \quad (6)$$

Next we use equation (1) and obtain ***v***^{2}

$$v^2 = v_0^2 + \frac{2E}{m} \quad (7)$$

we substitute into (6) and find

$$L = \frac{v_0^2}{g} + h + \frac{2E}{mg} \quad (8)$$

which we represent schematically as

$$L = a + \frac{b}{m} \quad (9)$$

Equation (9) gives the dependence of the throw length on the mass of the implement. Roughly stated, a large value of ***a*** would indicate a good acceleration in the circle, while a large value of ***b*** indicates a strong push. Validating equation (9) is far from an easy task. While elite throwers may train with heavier or lighter implements the precise data are not available. When competitions like the one that spurred this study are held, one has just the results for two different shot weights, which only allow to fix the parameters of (9). If we analyse the results of the introduction using (9) we find the following set of parameters for the athletes:

Name of Athlete	a (m)	b (kgm)
Hoffa	14.0	56.0
Majewski	12.7	59.6
Armstrong	12.4	60.0
Rodhe	11.6	65.2
Giza	12.9	42.2
Zbroszczyk	13.1	37.2

In addition to drawing comparisons on elite athletes, application of this model can be used with junior male throwers provided the existence of useful data. Even though competitions are held with lighter implements, the elite junior throwers do also train with the heavier ones and often participate at senior events. Unfortunately the data is scarce and only one complete set of performances was found, that of the new prodigy thrower Jacko Gill (NZL). His personal records, all of them from 2011, are 20.38m with a 7.25kg, 22.31m with a 6kg, and 24.45m with the 5kg shot[10]. Using the two heavier implements and equation (9) we obtain the following values for ***a*** = 11.1m and ***b*** = 67.2kgm. With these values one is able to extrapolate a performance of 24.54m with a 5kg shot. This value is in agreement with the value obtained for the actual throw (24.45m). It is interesting to note that this athlete also trains with a heavier shot (8kg), and there exists one testimonial of a 18.20m throw. Our expression (9) predicts a throw around 19.5m, which would not have been impossible for the athlete if that weight were used in competition. Another junior athlete for whom some results exist is for Krzysztof Brzozowski (POL)[12]. His best performance records in 2012 were 19.63m with a 7.25kg shot, 21.78m with a 6kg while with the 5kg shot his personal best was 23.23m dating back to 2010.

We start by obtaining the parameters ***a*** and ***b*** from the 2012 performances and find ***a*** = 9.3m and ***b*** = 74.8kgm. With use of equation (9) for a 5kg shot we find a distance of 24.3m perfectly compatible with the one meter shorter result recorded when the athlete was younger, since a junior thrower is expected to improve substantially in two years. Finally we examined the case of the two time world champion David Storl (GER)[13] who as a junior in 2009, recorded 20.43m with a 7.25kg shot, and 22.73m with a 6kg one. No performance with the 5kg shot appears to be known past 2007 but the use of (9) and the fitted parameters (***a*** = 9.4m and ***b*** = 80.0 kgm) give a prediction of 25.4m, which is not at all unreasonable, given the excellence of this athlete.

Discussion

While based on quite reasonable hypotheses, the equation relating the length of a throw to the mass of the shot is blatantly wrong to the eyes of a physicist. Given an expression like (9) one is naturally led to the question of what happens at the limits $m \to \infty$ and $m \to 0$. The answers obtained through use of equation (9) at both limits are wrong. For $m \to \infty$ equation (9) predicts $L = a$ (i.e. a non-zero result), while for $m \to 0$ it predicts an infinitely long throw. Does this mean that our model is wrong? Not at all, for the absurd results obtained at the two limits are possibly due to the fact that some basic assumptions are violated.

Let us examine first the $m \to \infty$ case. The tacit hypothesis in our model is that the thrower accelerates always to the same final velocity ***v***, independently of the shot's mass. This can only be true as long as this mass is small compared to the body mass of the thrower, let's say up to 10%. As the weight of the implement becomes substantial the hypothesis breaks down. In any case, given the constraints of the material world, the heaviest shot we could expect to have at our disposal without exceeding the 11-13cm diameter is one at around 20kg. Still it is expected that even so far from the $m \to \infty$ limit, a 20kg shot would alter significantly the dynamics of the acceleration phase.

Referring now to the $m \to 0$ limit, the tacit hypothesis here is that the limit to the velocity imparted to the shot is solely due to the inertia of the shot itself. However when the mass of the latter becomes very small the inertia of the body parts (essentially the arm and the hand) becomes far from negligible. Another complication stems from the fact that for light, fast-moving implements, the resistance of the air cannot be neglected anymore and the study must be redone afresh, including this effect. All in all we expect equation (9) to be certainly valid for 4kg shots and perhaps still valid down to 3kg while it will most probably break down at the 2kg level.

Having discussed the limitations of equation (9) as to the mass of the implement, it is now possible to present a better model, which has the merit of possessing correct $m \to \infty$ and $m \to 0$ limits, at the expense of only a moderate complication. First, we take into account the mass of the implement during the acceleration phase. This leads to an expression for v_0 given by

$$v_0^2 = \frac{2E_0}{m + m_0} \qquad (10)$$

where m_0 is the mass of the athlete. Second, we introduce the arm inertia of the athlete during the push phase, whereupon expression (7) becomes

$$v^2 = v_0^2 + \frac{2E}{m + f} \qquad (11)$$

Combining (10) and (11) we find an expression for d of the form

$$d = \frac{1}{gh}\left(\frac{E_0}{m + m_0} + \frac{E}{m + f}\right) \qquad (12)$$

which can be written schematically

$$d = \frac{p}{m + m_0} + \frac{q}{m + f} \qquad (13)$$

However, in order to compute the throw length L we must use the full expression (4), since d is not guaranteed to be a large quantity anymore. We remark here that a new parameter f has made its appearance. While we expect its value to be of the order of a few kg there is no easy way to fit it precisely, in particular in view of the paucity of results on which to make a fit. Still, an upper limit can be obtained by asking that the value of p be positive. This results in an f value being smaller than 6kg. Based on this assumption thus we decided to pursue our calculations by fixing f at 5kg. Fitting the performances of Gill for m = 7.25kg and 6kg we found the values p = 113kgm and q = 100kgm. These values lead to a prediction of 24.1m for a 5kg shot while for a 8kg the result is 19.4m. On an anecdotal level we may mention the 37m throw of Gill with a 1kg shot, a result very far from the prediction of equation (9), which turns out to be in nice agreement with the value of L, predicted for the set of p and q just obtained, which is 37.6m.

While the use of equation (4) leads to a somewhat complicated expression linking L to m, it is quite easy to visualize this dependence graphically (see Figure 1, next page). This graph is based on using the parameters for Gill.

Moreover once the graph is obtained it is easy to introduce a simple expression fitting the curve. We find

$$L = \frac{l}{m + k} \qquad (14)$$

with l = 286kgm and k = 6.5kg. One interesting result of this fit is that the elite throwers should be able to throw an osmium shot of the same diameter as the regulation one but of a weight slightly over 20kg at a distance of over 10m (Of course we do not think that anyone would stage a competition with such a precious implement).

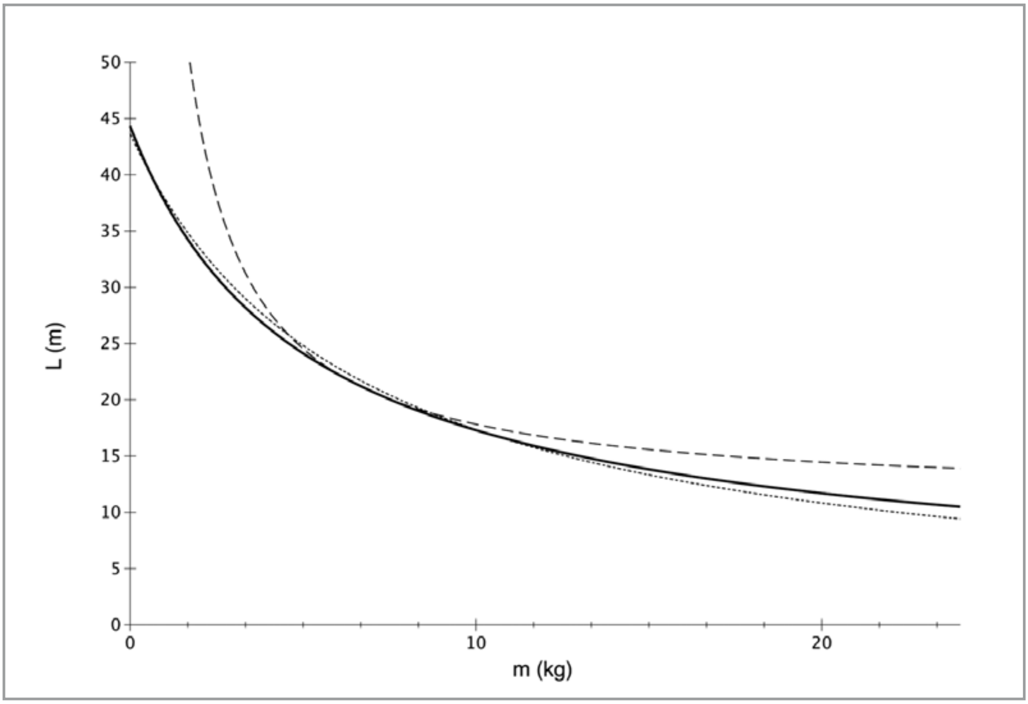

Figure 1: The thick line represents the results of the improved model, while the dashed one is obtained from equation (9). The dotted curve corresponds to the best fit with the simple analytical expression (14).

As an interesting aside of the analysis presented in this paper, one may wonder how this model might relate with female throwers. For instance, how far would Valerie Adams (NZL) throw a 7.25kg shot? I had the chance to exchange correspondence with Jean Pierre Egger, her coach, and I posed this question to him. It seems that in the past, Valerie, without special preparation, has thrown the 7.25kg shot 13m. Moreover, with a 5kg implement, she has thrown 18.24m, exactly 3m less than her best performance with the regulation 4kg shot. Let us try to analyse this data using equation (14).

We find readily ***I***=127kgm and ***k***=2kg. This leads to an estimate of 13.8m with a 7.25kg shot. While this performance is in agreement with the one mentioned above it is my feeling, as shared by her coach that she can perform better than this. In fact, the value of 2kg for ***k***, which plays a role of effective mass of the moving body parts (essentially the arm), is too low. If we assume a more realistic value of ***k*** = 4kg we find, using the performance of 21.24m for a 4kg shot, ***I*** = 170 kgm. Based on these values we can now predict that Adams should be able to throw the 5kg shot at 18.90m and the 7.25kg at just over 15m. This would be a respectable performance, coveted by many a male throwers at a regional level. Let us hope that such a test fits in the preparation of this athlete under the guidance of her coach and that some new data will be available in the near future confirming the present approach.

Conclusion

This paper provides a model of predicting the results of throwing a shot put with lighter, regulation and heavier shots. It is hoped that this model will provide coaches with another tool to aid in the training of their athletes. The prediction based on their current characteristics may provide a target for the athletes to aim for.

Acknowledgment

I am indebted to Jean Pierre Egger who kindly answered my questions, providing me with data on Valerie Adams but also offering a most useful insight on the coaching of throwers.

Please send all correspondence to:

Basil Grammaticos
grammati@paris7.jussieu.fr

REFERENCES

1. See report at http://www.iaaf.org/news/newsid=67597.html

2. SILVESTER, J. (2003). Complete Book of Throws. Champaign, IL: Human Kinetics.

3. LICHTENBERG, D.B & WILLS, J.G. (1978). Maximizing the range of the shot put. *Am. J. Phys.* 46, 546.

4. KUHLOW, A. & HEGER, W. (1975). Die Technik des Kugelstoßens der Manner bei den Olympischen Spielen 1972 in München, In: Beiheft zu Leistungssprt, 2

5. LINTHORNE, N.P. (2001). Optimum release angle in the shot put. *J. Sports Sci*. 19: 359.

6. DE LUCA, R. (2005). Shot-put kinematics. *Eur J Phys*, 26: 1031.

7. BARTONIETZ, K. & BORGSTROM, A. (1995). The throwing events at the World Championships in Athletics 1995, Goteborg - Technique of the world's best athletes. Part 1: shot put and hammer throw, *New Studies in Athletics*, 10: 43.

8. LUHTANEN, P.; BLOMQUIST, M. & VANTTINEN, T. (1997) A comparison of two elite putters using the rotational technique, New Studies in Athletics 12 (1997) 25.

9. LENZ, A. & RAPPL, F. (2010). The optimal angle of release in shot put. arXiv:1007.3689v2.

10. Data from http://www.iaaf.org/athletes/biographies/letter=g/country=nzl/athcode=250560/index.html

11. In the commentary of http://www.youtube.com/watch?v=znjA KCB7Clw

12. Data from http://www.iaaf.org/athletes/biographies/letter=b/country=pol/athcode=243599/index.html

13. Data from http://www.iaaf.org/athletes/biographies/letter=s/country=ger/athcode=227915/index.html